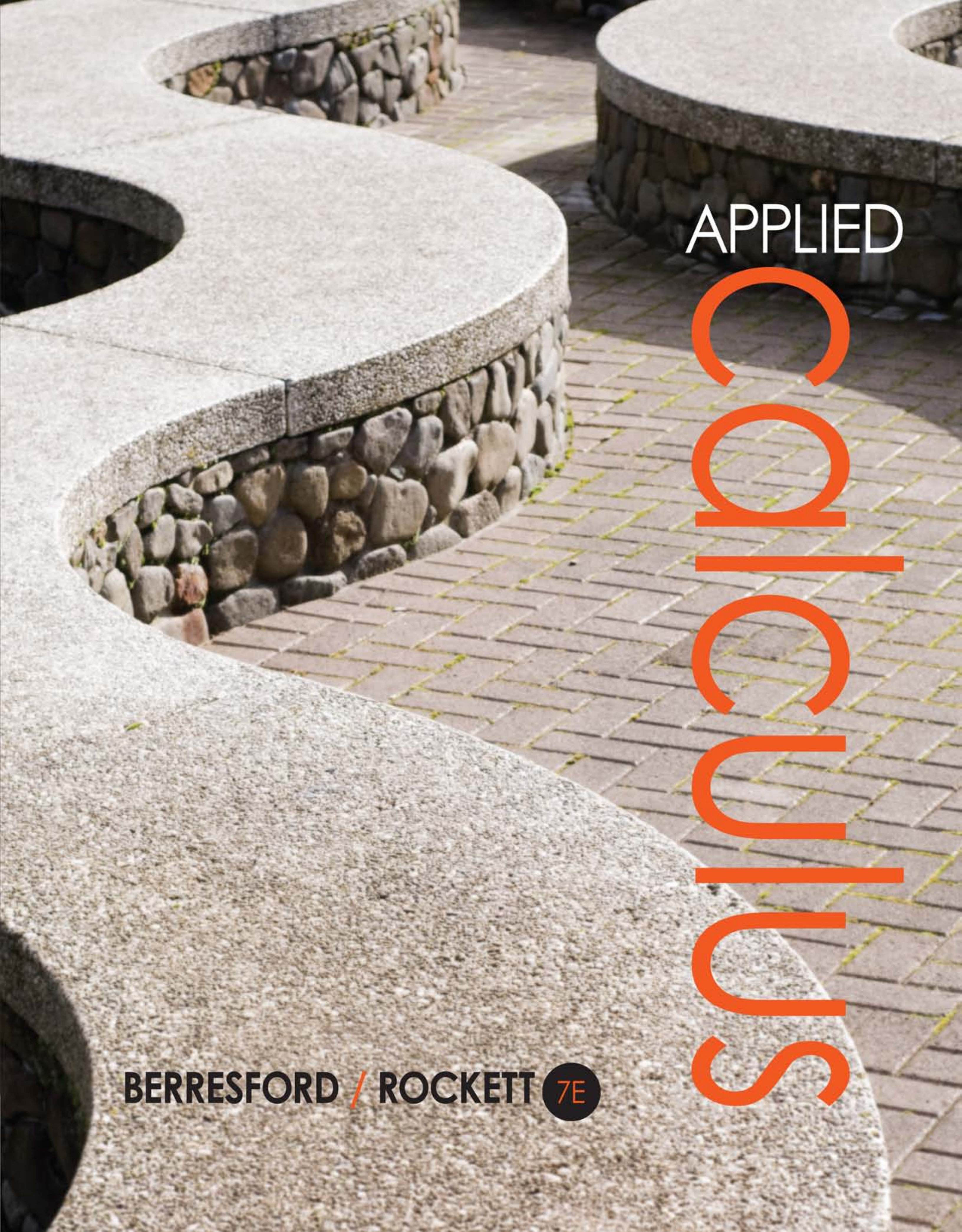
APPLIED
calculus
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7E

Topics of General Interest

applied Calculus

Seventh Edition

Geoffrey C. Berresford
Long Island University

Andrew M. Rockett
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Applied Calculus, Seventh Edition
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About the Cover

We were fortunate to have Lee Berresford's advice on the cover design. We thank her for her contributions.

WCN: 02-200-203

Library of Congress Control Number: 2014946320
Student Edition:

ISBN: 978-1-305-08531-2

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Printed in the United States of America
Print Number: 01 Print Year: 2014

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10 Sequences and Series

11 Probability

Overview

A scientific study of yawning found that more yawns occurred in calculus class than anywhere else.* This book hopes to remedy that situation. Rather than being another dry recitation of standard results, our presentation exhibits many of the fascinating and useful applications of mathematics in business, the sciences, and everyday life. Even beyond its utility, however, there is a beauty to calculus, and we hope to convey some of its elegance and simplicity.

This book is an introduction to calculus and its applications to the management, social, behavioral, and biomedical sciences, and other fields. The seven-chapter *Brief Applied Calculus* contains more than enough material for a one-semester course, and the eleven-chapter *Applied Calculus* contains additional chapters on trignometry, differential equations, sequences and series, and probability for a two-semester course. The only prerequisites are some knowledge of algebra, functions, and graphing, which are reviewed in Chapter 1 and in greater detail in the Algebra Review appendix.

ACCURATE AND ACCESSIBLE

Our foremost goal in writing these books has been to make the content as accessible to as many students as possible. Over time, we have introduced various features to address the changing needs of students as they learn the essential techniques and fundamental concepts of calculus. In order maintain students' interest and provide them with the most accurate and engaging textbook, we have been guided by the following principles.

- ***Informal Proofs*** Because this book is applied rather than theoretical, we have preferred intuitive and geometric justifications to formal proofs. We provide a justification or proof for every important mathematical idea. When proofs are given, they are correct and mathematically honest.
- ***Integration of Mathematics and Applications*** Every section has applications to motivate the mathematics being developed (see, for example, pages 27–28 and 119–120). There are no "pure math" sections.
- ***Rapid Start*** When learning something, it is best to begin doing it as soon as possible. Therefore, we keep the preliminary material brief so that students begin calculus without delay (in Section 2.2). An early start allows more time for interesting applications throughout the course.
- ***Just-in-Time Review*** Review material is placed just before it is used, where it is more likely to be remembered, rather than in lengthy early chapters that "review" material that was never mastered in the first place. Exponential and logarithmic functions are reviewed just before they are differentiated in Section 4.3, and the sine and cosine functions are reviewed just before they are differentiated in Section 8.3, as are the other trigonometric functions in Section 8.5.
- ***Continual Algebra Reinforcement*** Since many of today's students have weak algebra skills, which impede their understanding of calculus, examples have blue annotations in the right margin giving brief explanations of the steps (see, for example, page 88). For extra support, we also offer a Diagnostic Test (appearing before Chapter 1) to help students identify skills that may

*Ronald Baenninger, "Some Comparative Aspects of Yawning in Betta splendens, Homo sapiens, Panthera leo, and Papoi spinx," *Journal of Comparative Psychology* **101** (4).

need review along with a supplementary Algebra Review appendix for additional reference.

CHANGES IN THE SEVENTH EDITION

New Content

- Section 3.7 *Differentials, Approximations, and Marginal Analysis* is new in the seventh edition. This section is optional and can be omitted without loss of continuity.
- An Algebra Review appendix is keyed to parts of the text (see, for example, page 49).
- A Diagnostic Test has been added to help students identify skills that may need review. This test appears before Chapter 1. Complete solutions are given in the Algebra Review appendix.
- New material on parallel and perpendicular lines has been added to Section 1.1, *Real Numbers, Inequalities, and Lines*.
- New exercises have been added and over 100 updated (including all of the Wall Street financial exercises) with current real-world data and sources. New *Explorations and Excursions* exercises give further details or theoretical underpinnings of the topics in the main text.
- A new "What You'll Explore" paragraph on the opening page of each chapter previews the ideas and applications to come.

Enhanced Learning Support

- Throughout the text there are now LOOKING AHEAD and LOOKING BACK marginal notes that show connections between current material and past or future developments to unify students' understanding of calculus.
- New Take Note marginal prompts provide observations that simplify or clarify ideas.
- Newly added FOR MORE HELP and FOR HELP GETTING STARTED prompts point students to Examples or parts of the Algebra Review appendix for additional help.

Graphing Calculator

- The graphing calculator screens throughout the book are now in color, based on the TI-84 Plus C Silver Edition, although students can still use the TI-83 or TI-84 (regular or Plus) calculators and follow instructions provided to get corresponding black-and-white graphs.
- References to the Internet are now given for graphing calculator programs from sites such as `ticalc.org`. The programs may be used for Riemann sums (page 332), trapezoidal approximation (page 418), Simpson's rule (page 421), slope fields (pages 594, 596, and 614), and Euler's method (pages 634–635). For Newton's method the authors explain how the calculator may be used to perform the calculations directly with a few keystrokes (page 685). The graphing calculator programs from earlier editions are now available on the Student and the Instructor Companion Sites.

User's Guide

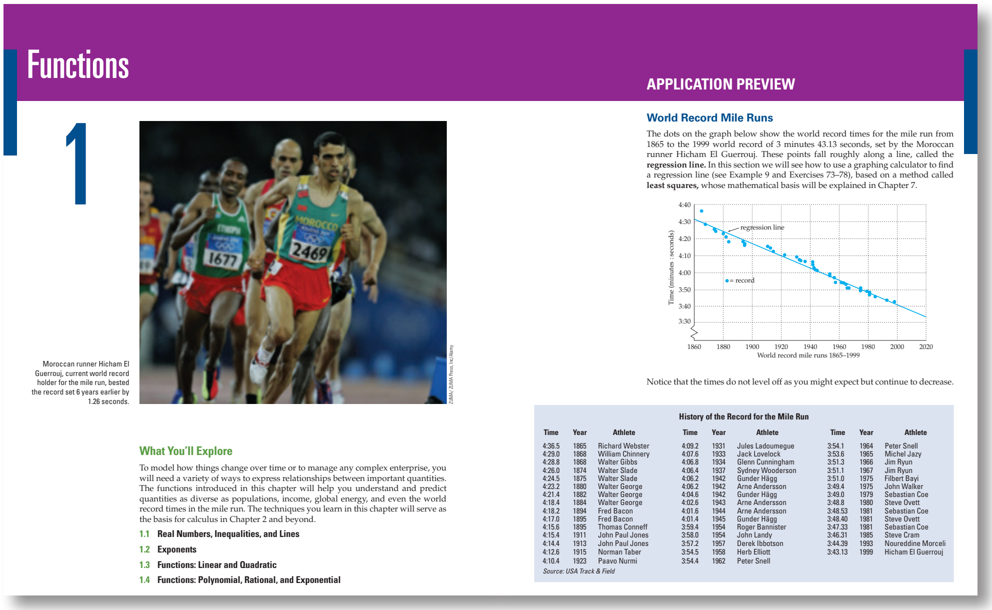
To get the most out of this book, familiarize yourself with the following features—all designed to increase your understanding and mastery of the material. These learning aids, together with any help available through your college, should make your encounter with calculus both successful and enjoyable.

APPLICATIONS

From archaeological finds to physics, from social issues to politics, the applications show that calculus is more than just manipulation of abstract symbols. Rather, it is a powerful tool that can be used to help understand and manage both the natural world and our activities in it.

Application Preview

Following each chapter opener, an Application Preview offers a "mathematics in your world" application. A page with further information on the topic and a related exercise number are often given.



Functions

1

Moroccan runner Hicham El Guerrouj, current world record holder for the mile run, bested the record set 6 years earlier by 1.26 seconds.

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What You'll Explore

To model how things change over time or to manage any complex enterprise, you will need a variety of ways to express relationships between important quantities. The functions introduced in this chapter will help you understand and predict quantities as diverse as populations, income, global energy, and even the world record times in the mile run. The techniques you learn in this chapter will serve as the basis for calculus in Chapter 2 and beyond.

1.1 Real Numbers, Inequalities, and Lines
1.2 Exponents
1.3 Functions: Linear and Quadratic
1.4 Functions: Polynomial, Rational, and Exponential

APPLICATION PREVIEW

World Record Mile Runs

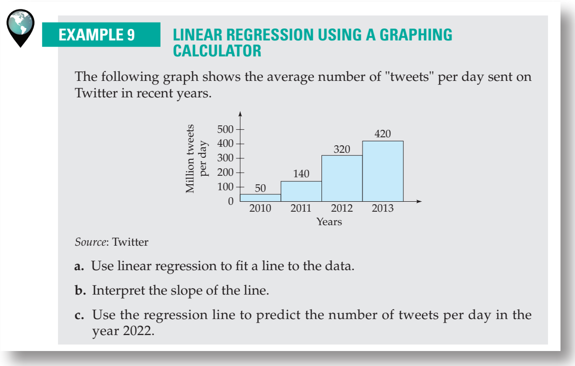
The dots on the graph below show the world record times for the mile run from 1865 to the 1999 world record of 3 minutes 43.13 seconds, set by the Moroccan runner Hicham El Guerrouj. These points fall roughly along a line, called the **regression line.** In this section we will see how to use a graphing calculator to find a regression line (see Example 9 and Exercises 73–78), based on a method called **least squares,** whose mathematical basis will be explained in Chapter 7.

Notice that the times do not level off as you might expect but continue to decrease.

History of the Record for the Mile Run

Time	Year	Athlete	Time	Year	Athlete	Time	Year	Athlete
4:36.5	1865	Richard Webster	4:09.2	1931	Jules Ladoumegue	3:54.1	1964	Peter Snell
4:29.0	1868	William Chinnery	4:07.6	1933	Jack Lovelock	3:53.6	1965	Michel Jazy
4:28.8	1868	Walter Gibbs	4:06.8	1934	Glenn Cunningham	3:51.3	1966	Jim Ryun
4:26.0	1874	Walter Slade	4:06.4	1937	Sydney Wooderson	3:51.1	1967	Jim Ryun
4:24.5	1875	Walter Slade	4:06.2	1942	Gunder Hägg	3:51.0	1975	Filbert Bayi
4:23.2	1880	Walter George	4:06.2	1942	Arne Andersson	3:49.4	1975	John Walker
4:21.4	1882	Walter George	4:04.6	1942	Gunder Hägg	3:49.0	1979	Sebastian Coe
4:18.4	1884	Walter George	4:02.6	1943	Arne Andersson	3:48.8	1980	Steve Ovett
4:18.2	1894	Fred Bacon	4:01.6	1944	Arne Andersson	3:48.53	1981	Sebastian Coe
4:17.0	1895	Fred Bacon	4:01.4	1945	Gunder Hägg	3:48.40	1981	Steve Ovett
4:15.6	1895	Thomas Conneff	3:59.4	1954	Roger Bannister	3:47.33	1981	Sebastian Coe
4:15.4	1911	John Paul Jones	3:58.0	1954	John Landy	3:46.31	1985	Steve Cram
4:14.4	1913	John Paul Jones	3:57.2	1957	Derek Ibbotson	3:44.39	1993	Noureddine Morceli
4:12.6	1915	Norman Taber	3:54.5	1958	Herb Elliott	3:43.13	1999	Hicham El Guerrouj
4:10.4	1923	Paavo Nurmi	3:54.4	1962	Peter Snell			

Source: USA Track & Field



EXAMPLE 9 LINEAR REGRESSION USING A GRAPHING CALCULATOR

The following graph shows the average number of "tweets" per day sent on Twitter in recent years.

Source: Twitter

a. Use linear regression to fit a line to the data.

b. Interpret the slope of the line.

c. Use the regression line to predict the number of tweets per day in the year 2022.

Diverse Applications

Along with an emphasis on business and biomedical sciences, a variety of other fields are represented throughout the text. Applications based on contemporary real-world data are denoted with an icon

GUIDED LEARNING SUPPORT

Annotations

To aid students' understanding of the solution steps within examples or to provide interpretations, blue annotations appear to the right of most mathematical formulas. Calculations presented within annotations provide explanations and justifications for the steps.

EXAMPLE 3 **DEPRECIATING AN ASSET**

A car worth \$30,000 depreciates in value by 40% each year. How much is it worth after 3 years?

Solution

The car loses 40% of its value each year, which is equivalent to an interest rate of *negative* 40%. The compound interest formula gives

$$30{,}000(1 - 0.40)^3 = \underbrace{30{,}000(0.60)^3}_{\text{Using a calculator}} = \$6480 \qquad P(1 + r/m)^{mt} \text{ with } P = 30{,}000,\ r = -0.40,\ m = 1, \text{ and } t = 3$$

The exponential function $f(x) = 30{,}000(0.60)^x$, giving the value of the car after x years of depreciation, is graphed on the left. Notice that a yearly loss of 40% means that 60% of the value is retained each year.

Be Careful

The "Be Careful" icon marks places where the authors help students avoid common errors.

4.3 Differentiation of Logarithmic and Exponential Functions 279

Be Careful Do *not* take the derivative of e^x by the Power Rule,

$$\frac{d}{dx}x^n = nx^{n-1}$$

The Power Rule applies to x^n, *a variable to a constant power,* while e^x is *a constant to a variable power.* The two types of functions are quite different, as their graphs show.

Looking Ahead
Looking Back

New in the 7e! These notes appear in the margins and show connections between current material and previous or future developments to solidify and unify understanding of calculus topics.

Compound Interest

For P dollars invested at annual interest rate r compounded m times a year for t years,

$$\begin{pmatrix}\text{Value after}\\ t \text{ years}\end{pmatrix} = P \cdot \left(1 + \frac{r}{m}\right)^{mt} \qquad \begin{aligned} r &= \text{annual rate}\\ m &= \text{periods per year}\\ t &= \text{number of years}\end{aligned}$$

LOOKING AHEAD On page 252 we will introduce a different kind of compound interest, where the compunding is done *continuously*.

For example, for monthly compounding we would use $m = 12$ and for daily compounding $m = 365$ (the number of days in the year).

EXAMPLE 3 **DIFFERENTIATING A LOGARITHMIC FUNCTION**

Find the derivative of $f(x) = \ln(x^4 - 1)^3$.

Solution

We need the rule for differentiating the natural logarithm of a function, together with the Generalized Power Rule [for differentiating $(x^4 - 1)^3$].

$$\frac{d}{dx}\ln(x^4 - 1)^3 = \frac{\frac{d}{dx}(x^4 - 1)^3}{(x^4 - 1)^3} \qquad \text{Using } \frac{d}{dx}\ln f = \frac{f'}{f}$$

$$= \frac{3(x^4 - 1)^2 4x^3}{(x^4 - 1)^3} \qquad \text{Using the Generalized Power Rule}$$

$$= \frac{12x^3}{x^4 - 1} \qquad \text{Dividing top and bottom by } (x^4 - 1)^2$$

FOR MORE HELP with simplifying expressions, see the Algebra Review appendix, pages B13–B14

Alternative Solution It is easier if we simplify first, using Property 8 of logarithms (see the inside back cover) to bring down the exponent 3:

$$\ln(x^4 - 1)^3 = 3\ln(x^4 - 1) \qquad \text{Using } \ln(M^P) = P \cdot \ln M$$

Now we differentiate the simplified expression:

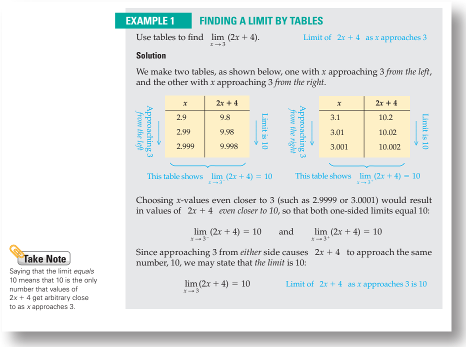
$$\frac{d}{dx}3\ln(x^4 - 1) = 3\frac{4x^3}{x^4 - 1} = \frac{12x^3}{x^4 - 1} \qquad \text{Same answer as before}$$

LOOKING BACK The properties of logarithms were stated on pages 262–263.

GUIDED LEARNING SUPPORT

Take Note

New in the 7/e! Appearing in the margins, these prompts include observations to help simplify or clarify ideas in the text.



EXAMPLE 1 FINDING A LIMIT BY TABLES

Use tables to find $\lim_{x \to 3}(2x+4)$. Limit of $2x+4$ as x approaches 3

Solution

We make two tables, as shown below, one with x approaching 3 *from the left*, and the other with x approaching 3 *from the right*.

x	$2x+4$
2.9	9.8
2.99	9.98
2.999	9.998

x	$2x+4$
3.1	10.2
3.01	10.02
3.001	10.002

Choosing x-values even closer to 3 (such as 2.9999 or 3.0001) would result in values of $2x+4$ *even closer to 10*, so that both one-sided limits equal 10:

$$\lim_{x \to 3^-}(2x+4) = 10 \quad \text{and} \quad \lim_{x \to 3^+}(2x+4) = 10$$

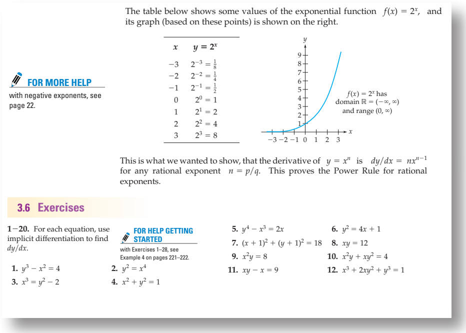
Since approaching 3 from *either* side causes $2x+4$ to approach the same number, 10, we may state that *the limit* is 10:

$$\lim_{x \to 3}(2x+4) = 10$$ Limit of $2x+4$ as x approaches 3 is 10

Take Note
Saying that the limit *equals* 10 means that 10 is the only number that values of $2x+4$ get arbitrary close to as x approaches 3.

For More Help
For Help Getting Started

New in the 7/e! These prompts appear within the margins of the text and end-of-section exercises. They direct students to Examples from within the text or parts of the Algebra Review appendix, as a refresher.



The table below shows some values of the exponential function $f(x) = 2^x$, and its graph (based on these points) is shown on the right.

x	$y = 2^x$
−3	$2^{-3} = \frac{1}{8}$
−2	$2^{-2} = \frac{1}{4}$
−1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$

FOR MORE HELP with negative exponents, see page 22.

This is what we wanted to show, that the derivative of $y = x^n$ is $dy/dx = nx^{n-1}$ for any rational exponent $n = p/q$. This proves the Power Rule for rational exponents.

3.6 Exercises

1–20. For each equation, use implicit differentiation to find dy/dx.

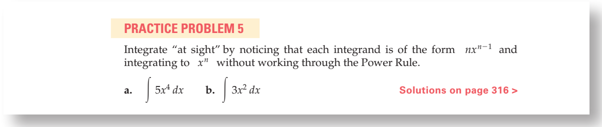
FOR HELP GETTING STARTED with Exercises 1–28, see Example 4 on pages 221–222.

1. $y^3 - x^2 = 4$ **2.** $y^2 = x^4$
3. $x^3 = y^2 - 2$ **4.** $x^2 + y^2 = 1$
5. $y^4 - x^3 = 2x$ **6.** $y^2 = 4x + 1$
7. $(x+1)^2 + (y+1)^2 = 18$ **8.** $xy = 12$
9. $x^2y = 8$ **10.** $x^2y + xy^2 = 4$
11. $xy - x = 9$ **12.** $x^3 + 2xy^2 + y^3 = 1$

PRACTICE AND PREPARE

Practice Problems

Students can check their understanding of a topic as they read the text or do homework by working out a Practice Problem. Complete solutions are found at the end of each section, just before the Section Summary.



PRACTICE PROBLEM 5

Integrate "at sight" by noticing that each integrand is of the form nx^{n-1} and integrating to x^n without working through the Power Rule.

a. $\int 5x^4\,dx$ **b.** $\int 3x^2\,dx$

Solutions on page 316 >

Exercises

The exercises that appear at the end of each section are graded from routine drills to significant applications. The *Applied Exercises* are labeled with general and specific titles so instructors can assign problems appropriate for the class. *Conceptual Exercises* develop intuitive insights to solve problems quickly and simply. *Explorations and Excursions* push students further. Just-in-time *Review Exercises* are found in selected sections. They recall skills previously learned that are relevant to content in an upcoming section (see, for example, page 355).

Although this may be used to *estimate* future costs (*about* \$15 for each additional unit), it does not mean that one additional unit will increase costs by exactly \$15, two more by exactly \$30, and so on, since the marginal rate usually changes as production increases. A marginal cost is only an *approximate* predictor of future costs.

2.5 Exercises

1–6. For each function, find:

a. $f'(x)$ **b.** $f''(x)$ **c.** $f'''(x)$ **d.** $f^{(4)}(x)$

1. $f(x) = x^4 - 2x^3 - 3x^2 + 5x - 7$
2. $f(x) = x^4 - 3x^3 + 2x^2 - 8x + 4$
3. $f(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5$
4. $f(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$
5. $f(x) = \sqrt{x^5}$
6. $f(x) = \sqrt{x^3}$

7–12. For each function, find: **a.** $f''(x)$ and **b.** $f''(3)$.

7. $f(x) = \dfrac{x-1}{x}$
8. $f(x) = \dfrac{x+2}{x}$

Applied Exercises

33. GENERAL: Velocity After t hours a freight train is $s(t) = 18t^2 - 2t^3$ miles due north of its starting point (for $0 \le t \le 9$).
- **a.** Find its velocity at time $t = 3$ hours.
- **b.** Find its velocity at time $t = 7$ hours.
- **c.** Find its acceleration at time $t = 1$ hour.

34. GENERAL: Velocity After t hours a passenger train is $s(t) = 24t^2 - 2t^3$ miles due west of its starting point (for $0 \le t \le 12$).
- **a.** Find its velocity at time $t = 4$ hours.
- **b.** Find its velocity at time $t = 10$ hours.
- **c.** Find its acceleration at time $t = 1$ hour.

35. GENERAL: Velocity A rocket can rise to a height of $h(t) = t^3 + 0.5t^2$ feet in t seconds. Find its velocity and acceleration 10 seconds after it is launched.

c. Find the acceleration at any time t. (This number is called the *acceleration due to gravity*.)

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Conceptual Exercises

47–50. Suppose that the quantity described is represented by a function $f(t)$ where t stands for time. Based on the description:

a. Is the first derivative positive or negative?
b. Is the second derivative positive or negative?

47. The temperature is dropping increasingly rapidly.
48. The economy is growing, but more slowly.
49. The stock market is declining, but less rapidly.
50. The population is growing increasingly fast.
51. True or False: If $f(x)$ is a polynomial of degree n, then $f^{(n+1)}(x) = 0$.

Explorations and Excursions

The following problems extend and augment the material presented in the text.

More About Higher-Order Derivatives

55. Find $\dfrac{d^{100}}{dx^{100}}(x^{100} - 4x^{99} + 3x^{50} + 6)$.

[*Hint:* You may use the "factorial" notation: $n! = n(n-1)\cdots 1$. For example, $3! = 3\cdot 2\cdot 1 = 6$.]

56. Find a general formula for $\dfrac{d^n}{dx^n}x^{-1}$.

[*Hint:* Calculate the first few derivatives and look for a pattern. You may use the "factorial" notation: $n! = n(n-1)\cdots 1$. For example, $3! = 3\cdot 2\cdot 1 = 6$.]

57. Verify the following formula for the *second* derivative of a product, where f and g are differentiable functions of x:

$$\frac{d^2}{dx^2}(f\cdot g) = f''\cdot g + 2f'\cdot g' + f\cdot g''$$

[*Hint:* Use the Product Rule repeatedly.]

58. Verify the following formula for the *third* derivative of a product, where f and g are differentiable functions of x:

$$\frac{d^3}{dx^3}(f\cdot g) = f'''\cdot g + 3f''\cdot g' + 3f'\cdot g'' + f\cdot g'''$$

[*Hint:* Differentiate the formula in Exercise 57 by the Product Rule.]

PRACTICE AND PREPARE

Section Summary

Found at the end of every section, summaries briefly state the main ideas of the section and provide study tools or reminders for students

3.7 Section Summary

For an *independent* variable x, the differential dx is any nonzero number. For the *dependent* variable $y = f(x)$, the differential is $dy = f'(x)\,dx$. Values for both x and dx must be known before dy can be evaluated.

The best linear approximation of a differentiable function $y = f(x)$ near x is the *tangent line approximation* given by

$$f(x + \Delta x) \approx f(x) + f'(x)\,dx \qquad (\Delta x = dx)$$

since $\Delta y \approx dy$. This approximation becomes more accurate for values of $\Delta x = dx$ closer to zero.

For a *dependent* variable y, the *error* Δy resulting from a measurement error Δx is sometimes called the *absolute* error, and may be approximated by the differential, $\Delta y \approx dy$. The *relative* error $\Delta y/y \approx dy/y$ compares the absolute error to the actual value, and is usually written as a percentage. Errors are sometimes called "changes" depending on the situation.

Marginals can be used to find approximations of revenue, cost, and profit (see page 234), and indicate how these quantities vary near a particular level of production.

Chapter Summary

Found at the end of every chapter, the Chapter Summary with Hints and Suggestions review the important developments of the chapter and give insights to unify the material to help students prepare for tests and exams.

2 Chapter Summary with Hints and Suggestions

Reading the text and doing the exercises in this chapter have helped you to master the following concepts and skills, which are listed by section (in case you need to review them) and are keyed to particular Review Exercises. Answers for all Review Exercises are given at the back of the book, and full solutions can be found in the Student Solutions Manual.

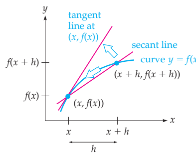
2.1 Limits and Continuity

- Find the limit of a function from tables. *(Review Exercises 1–2.)*
- Find left and right limits. *(Review Exercises 3–4.)*
- Find the limit of a function. *(Review Exercises 5–14.)*
- Determine whether a function is continuous or discontinuous. *(Review Exercises 15–22.)*

2.2 Rates of Change, Slopes, and Derivatives

- Find the derivative of a function from the *definition* of the derivative. *(Review Exercises 23–26.)*

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



$$MC(x) = C'(x) \qquad MR(x) = R'(x) \qquad MP(x) = P'(x)$$

- Find and interpret the derivative of a learning curve. *(Review Exercise 35.)*
- Find and interpret the derivative of an area or volume formula. *(Review Exercises 36–37.)*

2.4 The Product and Quotient Rules

- Find the derivative of a function using the Product Rule or Quotient Rule. *(Review Exercises 38–48.)*

$$\frac{d}{dx}(f \cdot g) = f' \cdot g + f \cdot g'$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g \cdot f' - g' \cdot f}{g^2}$$

- Find the tangent line to a curve at a given point. *(Review Exercise 49.)*
- Use differentiation to solve an applied problem and interpret the answer. *(Review Exercises 50–52.)*

$$MAC(x) = \frac{C(x)}{x}$$

$$MAR(x) = \frac{R(x)}{x}$$

$$MAP(x) = \frac{P(x)}{x}$$

Review Exercises and Chapter Test

Following the Chapter Summary are the Review Exercises and a Chapter Test. Selected questions from the Review Exercises are specially color-coded to indicate that they may be used as a practice Chapter Test. Both even and odd answers are supplied in the back of the book for students to check their proficiency.

2 Review Exercises and Chapter Test

○ indicates a Chapter Test exercise.

2.1 Limits and Continuity

1–2. Complete the tables and use them to find each limit (or state that it does not exist). Round calculations to three decimal places.

(1) **a.** $\lim_{x \to 2^-} (4x + 2)$

b. $\lim_{x \to 2^+} (4x + 2)$

c. $\lim_{x \to 2} (4x + 2)$

x	4x + 2
1.9	
1.99	
1.999	

x	4x + 2
2.1	
2.01	
2.001	

(2) **a.** $\lim_{x \to 0^-} \frac{\sqrt{x+1} - 1}{x}$

b. $\lim_{x \to 0^+} \frac{\sqrt{x+1} - 1}{x}$

c. $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$

x	$\frac{\sqrt{x+1} - 1}{x}$
−0.1	
−0.01	
−0.001	

x	$\frac{\sqrt{x+1} - 1}{x}$
0.1	
0.01	
0.001	

Cumulative Review

Cumulative Review questions appear after every three to four chapters, with all answers supplied in the back of the book.

1–3 Cumulative Review for Chapters 1–3

The following exercises review some of the basic techniques that you learned in Chapters 1–3. Answers to all of these cumulative review exercises are given in the answer section at the back of the book.

1. Find an equation for the line through the points $(-4, 3)$ and $(6, -2)$. Write your answer in the form $y = mx + b$.

2. Simplify $(\frac{4}{25})^{-1/2}$.

3. Find, correct to three decimal places: $\lim_{x \to 0} (1 + 3x)^{1/x}$.

4. For the function $f(x) = \begin{cases} 4x - 8 & \text{if } x < 3 \\ 7 - 2x & \text{if } x \geq 3 \end{cases}$

a. Draw its graph.

b. Find $\lim_{x \to 3^-} f(x)$.

c. Find $\lim_{x \to 3^+} f(x)$.

d. Find $\lim_{x \to 3} f(x)$.

e. Is $f(x)$ continuous or discontinuous, and if it is discontinuous, where?

5. Use the definition of the derivative, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, to find the derivative of $f(x) = 2x^2 - 5x + 7$.

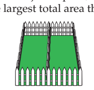
6. Find the derivative of $f(x) = 8\sqrt{x^3} - \frac{3}{x^2} + 5$.

14. Find the equation for the tangent line to the curve $y = \frac{4(x+3)}{\sqrt{x^2 + 3}}$ at $x = -1$.

15. Make sign diagrams for the first and second derivatives and draw the graph of the function $f(x) = x^3 - 12x^2 - 60x + 400$. Show on your graph all relative extreme points and inflection points.

16. Make sign diagrams for the first and second derivatives and draw the graph of the function $f(x) = \sqrt[3]{x^2} - 1$. Show on your graph all relative extreme points and inflection points.

17. A homeowner wishes to use 600 feet of fence to enclose two identical adjacent pens, as in the diagram below. Find the largest total area that can be enclosed.



18. A store can sell 12 telephone answering machines per day at a price of $200 each. The manager estimates that for each $10 price reduction she can sell 2 more per day. The answering machines cost the store $80

TECHNOLOGY

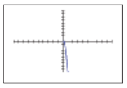
OPTIONAL! Using this book does not require a graphing calculator, but having one will enable you to do many problems more easily and as the same time deepen your understanding by allowing you to concentrate on concepts. The displays shown in the text are from the Texas Instruments TI-84 Plus *C* Silver Edition, except for a few from the TI-89, but any graphing calculator or computer may be used instead. For those who do not have a graphing calculator, the Explorations have been designed to be read for enrichment.

Similarly, if you have access to a computer, you may wish to do some of the Spreadsheet Explorations.

Graphing Calculator Explorations

To allow for optional use of the graphing calculator, these Explorations are boxed. Most can also be read simply for enrichment. Exercises and examples that are designed to be done with a graphing calculator are marked with an icon.

Graphing Calculator Exploration


on [−10, 10] by [−10, 10]

The graph of the function from Example 5, $y_1 = 9x - 20x^{3/2}$ (written in x instead of t for ease of entry), is shown on the left on the standard window [−10, 10] by [−10, 10]. This might lead you to believe, erroneously, that the function is maximized at the endpoint (0, 0).

a. Why does this graph not look like the graph at the end of the previous example? [*Hint:* Look at the scale.]

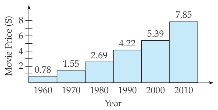
b. Can you find a window on which your graphing calculator will show a graph like the one at the end of the preceding solution?

This example illustrates one of the pitfalls of graphing calculators—the part of the curve where the "action" takes place may be entirely hidden in one pixel. Calculus, on the other hand, will *always* find the critical value, no matter where it is, and then a graphing calculator can be used to confirm your answer by showing the graph on an appropriate window.

Modeling

Selected application exercises feature regression capabilities of graphing calculators to fit curves to actual data.

90. BUSINESS: Movie Prices National average theater admissions prices for recent decades are shows in the following graph.



a. Number the bars with x-values 1–6 (so that x stands for *decades since 1950*) and use quadratic regression to fit a parabola to the data. State the regression function. [*Hint:* See Example 10.]

b. Use the regression function to predict movie prices in the years 2020 and 2030.

Source: Entertainment Weekly

Spreadsheet Explorations

Boxed for optional use, these explorations will enhance students' understanding of the material using Excel for those who prefer spreadsheet technology. See "Integrating Excel" on the next page for a list of exercises that can be done with Excel.

Spreadsheet Exploration

Another function that is not differentiable is $f(x) = x^{2/3}$. The following spreadsheet* calculates values of the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ at $x = 0$ for this function. Since $f(0) = 0$, the difference quotient at $x = 0$ simplifies to:

$$\frac{f(x+h) - f(x)}{h} = \frac{f(0+h) - f(0)}{h} = \frac{f(h)}{h} = \frac{h^{2/3}}{h} = h^{-1/3}$$

For example, cell **B5** evaluates $h^{-1/3}$ at $h = \frac{1}{1000}$ obtaining $(\frac{1}{1000})^{-1/3} = 1000^{1/3} = \sqrt[3]{1000} = 10$. Column **B** evaluates this different quotient for the *positive* values of h in column **A**, while column **E** evaluates it for the corresponding negative values of h in column **D**.

B5 = =A5^(-1/3)

	A	B	C	D	E
1	h	(f(0+h)-f(0))/h		h	(f(0+h)-f(0))/h
2	1.0000000	1.0000000		-1.0000000	-1.0000000
3	0.1000000	2.1544347		-0.1000000	-2.1544347
4	0.0100000	4.6415888		-0.0100000	-4.6415888
5	0.0010000	10.0000000		-0.0010000	-10.0000000
6	0.0001000	21.5443469		-0.0001000	-21.5443469
7	0.0000100	46.4158883		-0.0000100	-46.4158883
8	0.0000010	100.0000000		-0.0000010	-100.0000000
9	0.0000001	215.4434690		-0.0000001	-215.4434690

Column B: becoming large. Column E: becoming small.

Notice that the values in column **B** are becoming arbitrarily large, while the values in column **E** are becoming arbitrarily small, so the difference quotient does not approach a limit as $h \to 0$. This shows that the derivative of $f(x) = x^{2/3}$ at 0 *does not exist*, so the function $f(x) = x^{2/3}$ is *not differentiable* at $x = 0$.

INTEGRATING EXCEL

If you would like to use Excel or another spreadsheet software when working the exercises in this text, refer to the chart below. It lists exercises from many sections that you might find instructive to do with spreadsheet technology. If you would like help using Excel, please consider the *Excel Guide* available via CengageBrain.com.

Section	Suggested Exercises	Section	Suggested Exercises
1.1	59–78	6.1	60–64
1.2	103–110	6.2	65, 66, 68
1.3	69–82, 84–90	6.3	41–42
1.4	79–92	6.4	9–18, 27–37
2.1	77–78, 81–82	7.1	29–30, 38–42
2.2	9–16	7.2	47–48, 53–56
2.3	47–50	7.3	29–32
2.4	61–64	7.4	13–18, 27–32
2.5	45–46	7.5	29–36
2.6	65, 69	7.6	31–32, 35–36
2.7	11–12	7.7	41–42
3.1	68–71, 85	8.1	9–20
3.2	61–64	8.2	36–41
3.3	23–40, 52–54	8.3	73–80
3.4	23–24	8.4	49–54
3.5	20	8.5	13–16, 20–26
3.6	69–70	9.1	71
3.7	23–26	9.2	54
4.1	11–12, 47–51	9.3	36–38, 41–48
4.2	31–50	9.4	11–24, 27–30
4.3	97–99	10.1	49–59
4.4	38–39	10.2	9–12, 21–22
5.1	41–42	10.3	35–36
5.2	45–46, 55–58	10.4	11–24, 33–38
5.3	13–18, 83–88	11.1	17–18, 29–36
5.4	32, 35–36, 61, 69	11.2	37–41
5.5	31–32	11.3	23
5.6	77–78	11.4	21–26

SUPPLEMENTS

For the Student	For the Instructor
Student Solutions Manual ISBN: 978-1-305-10795-3 This manual contains fully worked-out solutions to all of the odd-numbered exercises in the text, giving students a way to check their answers and ensure that they took the correct steps to arrive at an answer.	**Complete Solutions Manual** This manual contains solutions to all exercises from the text including Chapter Review Exercises and Cumulative Reviews. It also contains two chapter-level tests for each chapter, one short-answer and one multiple choice, along with answers to each. This manual can be found on the Instructor Companion Site.
CengageBrain.com To access additional course materials, please visit **www.cengagebrain.com.** At the **CengageBrain.com** home page, search for the ISBN (from the back cover of your book) of your title using the search box at the top of the page. This will take you to the product page where these resources can be found.	**Instructor Companion Site** Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online via **www.cengage.com/login.** Access and download PowerPoint® presentations, images, solutions manual, and more.
Enhanced WebAssign® Instant Access Code: 978-1-285-85761-9 Printed Access Card: 978-1-285-85758-9 Enhanced WebAssign combines exceptional mathematics content with the most powerful online homework solution, WebAssign. It now includes QuickPrep content to review key precalculus content, available as a CoursePack of prebuilt assignments to assign at the beginning of the course or where needed most. Enhanced WebAssign engages students with immediate feedback, rich tutorial content, and an interactive, fully customizable eBook, the Cengage YouBook, helping students to develop a deeper conceptual understanding of their subject matter.	**Enhanced WebAssign®** Instant Access Code: 978-1-285-85761-9 Printed Access Card: 978-1-285-85758-9 Enhanced WebAssign combines exceptional mathematics content with the most powerful online homework solution, WebAssign. It now includes QuickPrep content to review key precalculus content, available as a CoursePack of prebuilt assignments to assign at the beginning of the course or where needed most. Enhanced WebAssign engages students with immediate feedback, rich tutorial content, and an interactive, fully customizable eBook, the Cengage YouBook, helping students to develop a deeper conceptual understanding of their subject matter. Visit **www.cengage.com/ewa** to learn more.
	Cengage Learning Testing Powered by Cognero® Instant Access Code: 978-1-305-11229-2 Cognero is a flexible, online system that allows you to author, edit, and manage test bank content, create multiple test versions in an instant, and deliver tests from your LMS, your classroom or wherever you want. This is available online via **www.cengage.com/login**.

ACKNOWLEDGMENTS

We are indebted to many people for their useful suggestions, conversations, and correspondence during the writing and revising of this book. We thank Chris and Lee Berresford, Anne Burns, Richard Cavaliere, Ruch Enoch, Theordore Faticoni, Jeff Goodman, Susan Halter, Brita and Ed Immergut, Ethel Matin, Gary Patric, Shelly Rothman, Charlene Russert, Stuart Saal, Bob Sickles, Michael Simon, John Stevenson, and all of our "Math 6" students at C.W. Post for serving as proofreaders and critics over many years.

We had the good fortune to have had the support of expert editorial, production, and marketing colleagues at Cengage Learning: Richard Stratton; Rita Lombard; Erin Brown; Jennifer Cordoba; Jessica Rasile; Jill Quinn; Linda May; and Julie Schuster. We also express our gratitude to the many others at Cengage Learning who made important contributions, too numerous to mention. We are especially grateful to Magdalena Luca who worked to ensure the accuracy of the seventh edition.

We also wish to acknowledge Christi Verity and Aldena Calden (UMass, Amherst) for their contributions on the solutions manuals. Very special thanks also go to Lee Berresford for her contributions on the cover design.

The following reviewers and readers have contributed greatly to the development of the seventh edition of this text:

Haya Adner	*Queensborough Community College*
Kimberly Benien	*Wharton County Junior College*
Mark Billiris	*St. Petersburg College*
Lynn Cade	*Pensacola State College*
Seo-eun Choi	*Arkansas State University*
Cindy Dickson	*College of Southern Idaho*
Susan Howell	*University of Southern Mississippi*
Magdalena Luca	*MCPHS University*
Kevin Lynch	*Northeast State Community College*
Cornelius Nelan	*Quinnipiac University*
Victor Swaim	*Southeastern Louisiana University*
William Veczko	*St. Johns River State College*

We also thank the reviewers of recent editions:

Frederick Adkins, *Indiana University of Pennsylvania*; David Allen, *Iona College*; Joel M. Berman, *Valencia Community College*; John A. Blake, *Oakwood College*; Dave Bregenzer, *Utah State University*; Kelly Brooks, *Pierce College*; Donald O. Clayton, *Madisonville Community College*; Charles C. Clever, *South Dakota State University*; Julane Crabtree, *Johnson Community College*; Dale L. Craft, *South Florida Community College*; Kent Craghead, *Colby Community College*; Biswa Datta, *Northern Illinois University*; Lloyd David, *Montreat College*; Allan Donsig, *University of Nebraska—Lincoln*; Sally Edwards, *Johnson Community College*; Frank Farris, *Santa Clara University*; Brad Feldser, *Kennesaw State University*; Daria Filippova, *Bowling Green State University;* Abhay Gaur, *Duquesne University*; Jerome Goldstein, *University of Memphis*; John Haverhals, *Bradley University*; John B. Hawkins, *Georgia Southern University*; Randall Helmstutler, *University of Virginia*; Susan Howell, *University of Southern Mississippi;* Heather Hulett, *University of Wisconsin—La Crosse*; David Hutchison, *Indiana State University*; Dan Jelsovsky, *Florida Southern College*; Alan S. Jian, *Solano Community College*; Dr. Hilbert Johs, *Wayne State College*; Hideaki Kaneko, *Old Dominion University*; John Karloff, *University of North Carolina*; Susan Kellicut, *Seminole Community College;* Todd King, *Michigan Technical University;* JoAnn Kump, *West Chester University;* Richard Leedy, *Polk Community College*; Michael Longfritz, *Rensselear Polytechnic Institute*; Dr. Hank Martel, *Broward Community College*; Kimberly McGinley Vincent, *Washington State University*; Donna Mills, *Frederick Community College*; Pat Moreland, *Cowley College*; Sanjay Mundkur, *Kennesaw State University*; Sue Neal, *Wichita State University*; Cornelius Nelan, *Quinnipiac University*; David Parker, *Salisbury University*; Shahla Peterman, *University of Missouri—Rolla*; Susan Pfiefer, *Butler Community College*; Daniel Plante, *Stetson University*; Brooke Quinlan, *Hillsborough Community College;* Catherine A. Roberts, *University of Rhode Island*; George W. Schultz, *St. Petersburg College*; Larry Small, *Pierce College;* Paul H. Stanford, *University of Texas—Dallas*; Xingping Sun, *Missouri State University*; Jill Van Valkenburg, *Bowling Green State University*; Jaak Vilms, *Colorado State University*; Erica Voges, *New Mexico State University*; Jane West, *Trident Technical College*; Elizabeth White, *Trident Technical College*; Kenneth J. Word, *Central Texas College*; Wen-Qing Xu, *California State University—Long Beach.*

Finally, and most importantly, we thank our wives, Barbara and Kathryn, for their encouragement and support.

COMMENTS WELCOMED

With the knowledge that any book can always be improved, we welcome corrections, constructive criticisms, and suggestions from every reader.

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DIAGNOSTIC TEST

Are you ready to study calculus?

Algebra is the language in which we express the ideas of calculus. Therefore, to understand calculus and express its ideas with precision, you need to know some algebra.

If you are comfortable with the algebra covered in the following problems, you are ready to begin your study of calculus. If not, turn to the *Algebra Review* appendix beginning on page B1 and review the *Complete Solutions* to these problems, and continue reading the other parts of the Appendix that cover anything that you do not know.

Problems

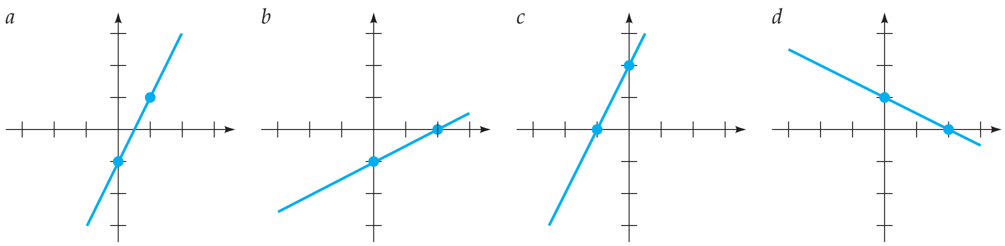
1. True or False? $\frac{1}{2} < -3$

2. Express $\{x \mid -4 < x \leq 5\}$ in interval notation.

3. What is the slope of the line through the points $(6, -7)$ and $(9, 8)$?

4. On the line $y = 3x + 4$, what value of Δy corresponds to $\Delta x = 2$?

5. Which sketch shows the graph of the line $y = 2x - 1$?



6. True of False? $\left(\frac{\sqrt{x}}{y}\right)^{-2} = \frac{y^2}{x}$

7. Find the zeros of the function $f(x) = 9x^2 - 6x - 1$.

8. Expand and simplify $x(8 - x) - (3x + 7)$.

9. What is the domain of $f(x) = \frac{x^2 - 3x + 2}{x^3 + x^2 - 6x}$?

10. Find the difference quotient $\frac{f(x+h) - f(x)}{h}$ for $f(x) = x^2 - 5x$.

Answers

1. False
2. $(-4, 5]$
3. 5
4. 6
5. *a*
6. True
7. $x = \frac{1 \pm \sqrt{2}}{3}$
8. $-x^2 + 5x - 7$
9. $\{x \mid x \neq -3, x \neq 0, x \neq 2\}$
10. $2x - 5 + h$

Functions

1

ZUMA/ ZUMA Press, Inc/Alamy

Moroccan runner Hicham El Guerrouj, current world record holder for the mile run, bested the record set 6 years earlier by 1.26 seconds.

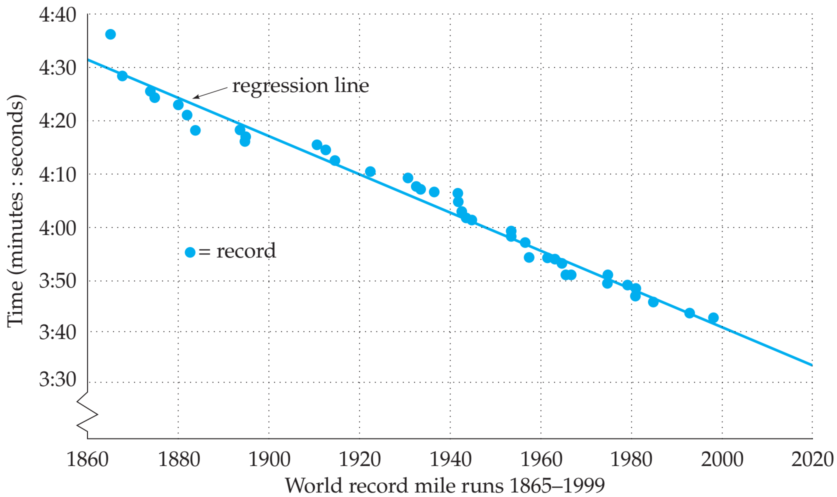
What You'll Explore

To model how things change over time or to manage any complex enterprise, you will need a variety of ways to express relationships between important quantities. The functions introduced in this chapter will help you understand and predict quantities as diverse as populations, income, global energy, and even the world record times in the mile run. The techniques you learn in this chapter will serve as the basis for calculus in Chapter 2 and beyond.

APPLICATION PREVIEW

World Record Mile Runs

The dots on the graph below show the world record times for the mile run from 1865 to the 1999 world record of 3 minutes 43.13 seconds, set by the Moroccan runner Hicham El Guerrouj. These points fall roughly along a line, called the **regression line.** In this section we will see how to use a graphing calculator to find a regression line (see Example 9 and Exercises 73–78), based on a method called **least squares,** whose mathematical basis will be explained in Chapter 7.



Notice that the times do not level off as you might expect but continue to decrease.

History of the Record for the Mile Run

Time	Year	Athlete	Time	Year	Athlete	Time	Year	Athlete
4:36.5	1865	Richard Webster	4:09.2	1931	Jules Ladoumegue	3:54.1	1964	Peter Snell
4:29.0	1868	William Chinnery	4:07.6	1933	Jack Lovelock	3:53.6	1965	Michel Jazy
4:28.8	1868	Walter Gibbs	4:06.8	1934	Glenn Cunningham	3:51.3	1966	Jim Ryun
4:26.0	1874	Walter Slade	4:06.4	1937	Sydney Wooderson	3:51.1	1967	Jim Ryun
4:24.5	1875	Walter Slade	4:06.2	1942	Gunder Hägg	3:51.0	1975	Filbert Bayi
4:23.2	1880	Walter George	4:06.2	1942	Arne Andersson	3:49.4	1975	John Walker
4:21.4	1882	Walter George	4:04.6	1942	Gunder Hägg	3:49.0	1979	Sebastian Coe
4:18.4	1884	Walter George	4:02.6	1943	Arne Andersson	3:48.8	1980	Steve Ovett
4:18.2	1894	Fred Bacon	4:01.6	1944	Arne Andersson	3:48.53	1981	Sebastian Coe
4:17.0	1895	Fred Bacon	4:01.4	1945	Gunder Hägg	3:48.40	1981	Steve Ovett
4:15.6	1895	Thomas Conneff	3:59.4	1954	Roger Bannister	3:47.33	1981	Sebastian Coe
4:15.4	1911	John Paul Jones	3:58.0	1954	John Landy	3:46.31	1985	Steve Cram
4:14.4	1913	John Paul Jones	3:57.2	1957	Derek Ibbotson	3:44.39	1993	Noureddine Morceli
4:12.6	1915	Norman Taber	3:54.5	1958	Herb Elliott	3:43.13	1999	Hicham El Guerrouj
4:10.4	1923	Paavo Nurmi	3:54.4	1962	Peter Snell			

Source: USA Track & Field

The equation of the regression line is $y = -0.356x + 257.44$, where x represents years after 1900 and y is the time in seconds. The regression line can be used to predict the world mile record in future years. Notice that the most recent world record would have been predicted quite accurately by this line, since the rightmost dot falls almost exactly on the line.

Linear trends, however, must not be extended too far. The downward slope of this line means that it will eventually "predict" mile runs in a fraction of a second, or even in *negative* time (see Exercises 59 and 60 on pages 17–18). *Moral:* In the real world, linear trends do not continue indefinitely. This and other topics in "linear" mathematics will be developed in Section 1.1.

1.1 Real Numbers, Inequalities, and Lines

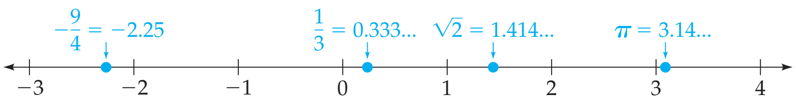
Introduction

Quite simply, *calculus is the study of rates of change*. We will use calculus to analyze rates of inflation, rates of learning, rates of population growth, and rates of natural resource consumption.

In this first section we will study **linear** relationships between two variable quantities—that is, relationships that can be represented by **lines.** In later sections we will study **nonlinear** relationships, which can be represented by **curves.**

Real Numbers and Inequalities

In this book the word "number" means **real number,** a number that can be represented by a point on the number line (also called the **real line**).



The *order* of the real numbers is expressed by **inequalities.** For example, $a < b$ means "a is to the *left* of b" or, equivalently, "b is to the *right* of a."

Inequalities

Inequality	In Words	Brief Examples
$a < b$	a is less than (smaller than) b	$3 < 5$
$a \leq b$	a is less than or equal to b	$-5 \leq -3$
$a > b$	a is greater than (larger than) b	$\pi > 3$
$a \geq b$	a is greater than or equal to b	$2 \geq 2$

The inequalities $a < b$ and $a > b$ are called **strict inequalities,** and $a \leq b$ and $a \geq b$ are called **nonstrict inequalities.**

IMPORTANT NOTE Throughout this book are many **Practice Problems**—short questions designed to check your understanding of a topic before moving on to new material. Full solutions are given at the end of the section. Solve the following Practice Problem and then check your answer.

PRACTICE PROBLEM 1

Which number is smaller: $\frac{1}{100}$ or $-1{,}000{,}000$?

Solution on page 15 >

Multiplying or dividing both sides of an inequality by a negative number reverses the direction of the inequality:

$$-3 < 2 \quad \text{but} \quad 3 > -2 \qquad \text{Multiplying by } -1$$

A **double inequality,** such as $a < x < b$, means that *both* the inequalities $a < x$ and $x < b$ hold. The inequality $a < x < b$ can be interpreted graphically as "x is between a and b."

$a < x < b$

LOOKING AHEAD

Sets and intervals will be important on page 33 when we define *domains* of functions.

Sets and Intervals

Braces { } are read "the set of all" and a **vertical bar** | is read "such that."

EXAMPLE 1 INTERPRETING SETS

The set of all

a. $\{ x \mid x > 3 \}$ means "the set of all x such that x is greater than 3."

Such that

b. $\{ x \mid -2 < x < 5 \}$ means "the set of all x such that x is between -2 and 5."

PRACTICE PROBLEM 2

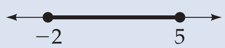
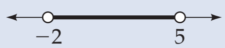
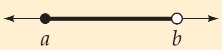
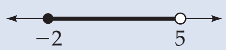
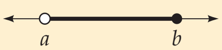
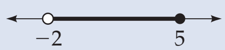
a. Write in set notation "the set of all x such that x is greater than or equal to -7."

b. Express in words: $\{ x \mid x < -1 \}$.

Solution on page 15 >

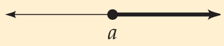
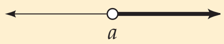
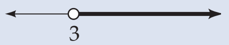
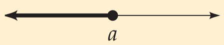
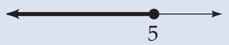
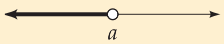
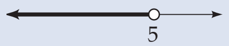
The set $\{ x \mid 2 \le x \le 5 \}$ can be expressed in **interval notation** by enclosing the endpoints 2 and 5 in **square brackets,** [2, 5], to indicate that the endpoints are *included*. The set $\{ x \mid 2 < x < 5 \}$ can be written with **parentheses,** (2, 5), to indicate that the endpoints 2 and 5 are *excluded*. An interval is **closed** if it includes both endpoints, and **open** if it includes neither endpoint. The four types of intervals are shown below: a **solid dot** • on the graph indicates that the point is *included* in the interval; a **hollow dot** ∘ indicates that the point is *excluded*.

Finite Intervals

Interval Notation	Set Notation	Graph	Type	Brief Examples	
$[a, b]$	$\{ x \mid a \le x \le b \}$		Closed (includes endpoints)	$[-2, 5]$	
(a, b)	$\{ x \mid a < x < b \}$		Open (excludes endpoints)	$(-2, 5)$	
$[a, b)$	$\{ x \mid a \le x < b \}$		Half-open or half-closed	$[-2, 5)$	
$(a, b]$	$\{ x \mid a < x \le b \}$			$(-2, 5]$	

An interval may extend infinitely far to the *right* (indicated by the symbol ∞ for **infinity**) or infinitely far to the *left* (indicated by $-\infty$ for **negative infinity**). Note that ∞ and $-\infty$ are not numbers but are merely symbols to indicate that the interval extends endlessly in that direction. The infinite intervals in the following box are said to be **closed** or **open** depending on whether they *include* or *exclude* their single endpoint.

Infinite Intervals

Interval Notation	Set Notation	Graph	Type	Brief Examples	
$[a, \infty)$	$\{ x \mid x \geq a \}$		Closed	$[3, \infty)$	
(a, ∞)	$\{ x \mid x > a \}$		Open	$(3, \infty)$	
$(-\infty, a]$	$\{ x \mid x \leq a \}$		Closed	$(-\infty, 5]$	
$(-\infty, a)$	$\{ x \mid x < a \}$		Open	$(-\infty, 5)$	

We use *parentheses* rather than square brackets with ∞ and $-\infty$ since they are not actual numbers.

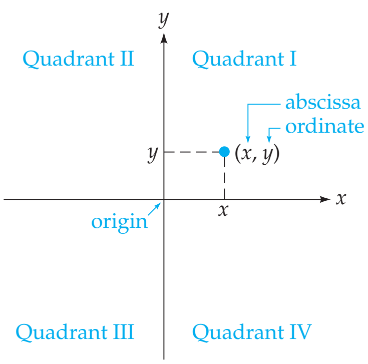
The interval $(-\infty, \infty)$ extends infinitely far in *both* directions (meaning the entire real line) and is also denoted by $\mathbb{R}$ (the set of all real numbers).

$\mathbb{R} = (-\infty, \infty)$

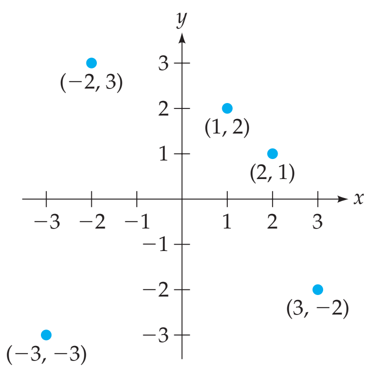
Cartesian Plane

Two real lines or **axes,** one horizontal and one vertical, intersecting at their zero points, define the **Cartesian plane.*** The point where they meet is called the **origin.** The axes divide the plane into four **quadrants,** I through IV, as shown below.

Any point in the Cartesian plane can be specified uniquely by an ordered pair of numbers (x, y); x, called the **abscissa** or ***x*-coordinate,** is the number on the horizontal axis corresponding to the point; y, called the **ordinate** or ***y*-coordinate,** is the number on the vertical axis corresponding to the point.



The Cartesian plane



The Cartesian plane with several points. Order matters: (1, 2) is not the same as (2, 1)

LOOKING AHEAD

We will use the Δ notation again on page 95.

Lines and Slopes

The symbol Δ (read "delta," the Greek letter D) means "the change in." For any two points (x_1, y_1) and (x_2, y_2) we define

*So named because it was originated by the French philosopher and mathematician René Descartes (1596–1650). Following the custom of the day, Descartes signed his scholarly papers with his Latin name Cartesius, hence "Cartesian" plane.

$\Delta x = x_2 - x_1$ — The change in x is the difference in the x-coordinates

$\Delta y = y_2 - y_1$ — The change in y is the difference in the y-coordinates

Any two distinct points determine a line. A nonvertical line has a **slope** that measures the *steepness* of the line, and is defined as *the change in y divided by the change in x* for any two points on the line.

Take Note

One of the main purposes of calculus is to extend the concept of slope from lines to *curves*.

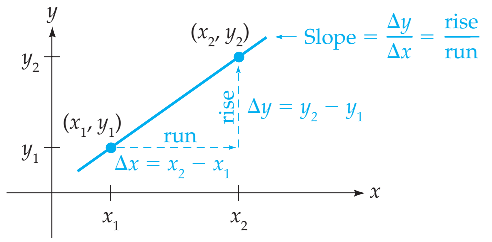
Slope of Line Through (x_1, y_1) and (x_2, y_2)

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope is the change in y over the change in x $(x_2 \neq x_1)$

Be Careful In slope, the x-values go in the *denominator*.

The changes Δy and Δx are often called, respectively, the "rise" and the "run," with the understanding that a negative "rise" means a "fall." Slope is then "rise over run."



EXAMPLE 2 FINDING SLOPES AND GRAPHING LINES

Find the slope of the line through each pair of points, and graph the line.

a. (2, 1), (3, 4) **b.** (2, 4), (3, 1)

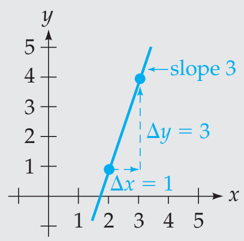
c. (−1, 3), (2, 3) **d.** (2, −1), (2, 3)

Solution

We use the slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ for each pair $(x_1, y_1), (x_2, y_2)$.

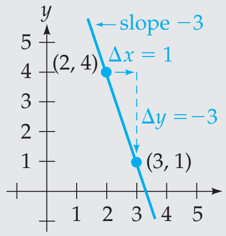
a. For (2, 1) and (3, 4) the slope is

$$\frac{4 - 1}{3 - 2} = \frac{3}{1} = 3.$$

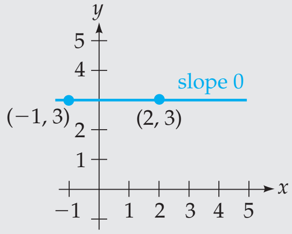


b. For (2, 4) and (3, 1) the slope is

$$\frac{1 - 4}{3 - 2} = \frac{-3}{1} = -3.$$

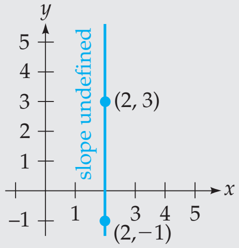


c. For $(-1, 3)$ and $(2, 3)$ the slope is $\dfrac{3-3}{2-(-1)} = \dfrac{0}{3} = 0.$



d. For $(2, -1)$ and $(2, 3)$ the slope is *undefined:*

$$\frac{3-(-1)}{2-2} = \frac{4}{0}.$$



Notice in the preceding graphs that when the x-coordinates are the same [as in part (d)], the line is *vertical*, and when the y-coordinates are the same [as in part (c)], the line is *horizontal*.

If $\Delta x = 1$, as in Examples 2a and 2b, then the slope is just the "rise," giving an alternative definition for slope:

$$\text{Slope} = \begin{pmatrix} \text{Amount that the line rises} \\ \text{when } x \text{ increases by 1} \end{pmatrix}$$

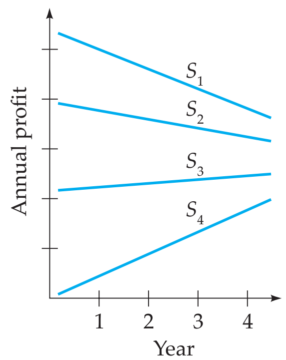
PRACTICE PROBLEM 3

A company president is considering four different business strategies, called S_1, S_2, S_3, and S_4, each with different projected future profits. The graph on the right shows the annual projected profit for the first few years for each of the strategies.

Which strategy yields:

a. the highest projected profit in year 1?
b. the highest projected profit in the long run?

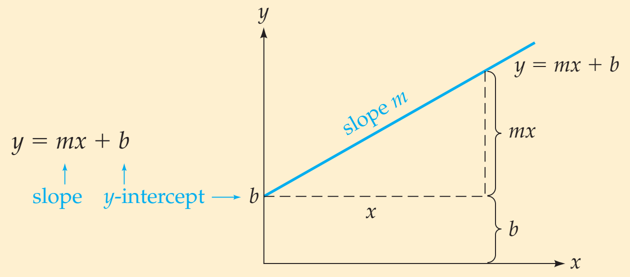
Solutions on page 15 >



Equations of Lines

The point where a nonvertical line crosses the y-axis is called the ***y*-intercept** of the line. The y-intercept can be given either as the y-coordinate b or as the point $(0, b)$. Such a line can be expressed very simply in terms of its slope and y-intercept, representing the points by variable coordinates (or "variables") x and y.

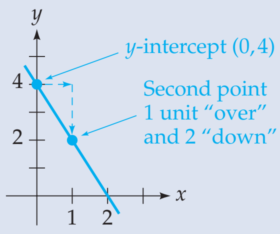
Slope-Intercept From of a Line

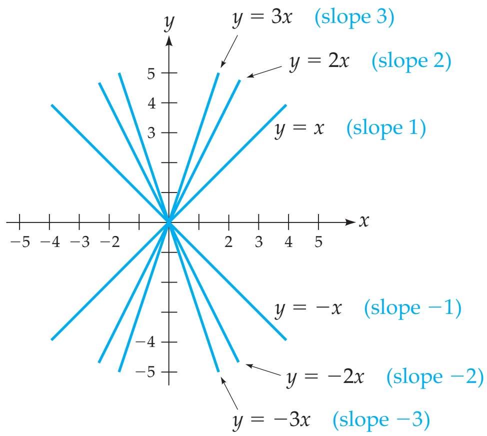


Brief Example

For the line with slope -2 and y-intercept 4:

$$y = -2x + 4$$





For lines through the origin, the equation takes the particularly simple form, $y = mx$ (since $b = 0$), as illustrated on the left.

The most useful equation for a line is the *point-slope form*.

Point-Slope Form of a Line

$$y - y_1 = m(x - x_1)$$

(x_1, y_1) = point on the line
m = slope

This form comes directly from the slope formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ by replacing x_2 and y_2 by x and y, and then multiplying each side by $(x - x_1)$. It is most useful when you know the slope of the line and a point on it.